THE NECESSARY CONDITIONS FOR STABLE

COMBUSTION OF A POWDER IN A HALF-CLOSED CHAMBER

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The low-frequency stability of the steady-state combustion cycle is investigated in linear approximation for a powder in a half-closed chamber, taking account of incompleteness of combustion, heat losses at the walls, and dynamic erosion. The necessary conditions for stable combustion are obtained. Qualitative conclusions are drawn of the incompleteness of the chemical reactions taking place and of the destabilizing effect of thermal losses and erosion on the combustion process. Only two out of three possible steady-state mechanisms are stable. The existence of limits of combustion with respect to pressure is shown.

1. Statement of the Problem

Theoretically, the stability of the steady-state combustion of a powder in a half-closed chamber, on the assumption of completeness of the chemical reactions, was studied in [1-4], etc. At reduced pressures, the heat of combustion is considerably different from its limiting value. Figure 1 (curve 1), copied from [5], shows the experimental dependence of the heat of combustion of nitroglycerine powder HES 4016 on the pressure. Reduction of the heat release can lead to a reduced temperature and pressure in the chamber and can exert a considerable effect on the stability of the combustion cycle.

In accordance with [5], we shall assume that under steady-state conditions, the heat of the chemical reactions Q is a function of the pressure p

$$Q^\circ = Q(p^\circ) \tag{1.1}$$

which is found, for example, by suitable interpolation of the experimental data (here and in the future the quantities for steady-state conditions are denoted by a degree symbol). In particular, the relations similar to that shown in Fig. 1 can be represented by functions of the type

$$Q(p^\circ) = \frac{Q_1 - [Q_1 - Q_0(1+g)]\exp[-L(p^\circ - d_0)]}{1 + g\exp[-L(p^\circ - d_0)]} \tag{1.2}$$

where

$$(Q_0 = Q(p^\circ = d_0), \quad Q_1 = Q(p^\circ \to \infty))$$

L and g are constants (L > 0, g > 1). The experimental data for HES 4016 powder (curve 1 in Fig. 1) are described with satisfactory accuracy by formula (1.2) with the following parameter values:

$$Q_1 = 1140 \text{ cal/g}, \quad Q_0 = 510 \text{ cal/g}, \quad d_0 = 14 \text{ kgf/cm}^2, L = 0.2 \text{ cm}^2/\text{kgf},$$
$$g = 24.$$

We shall represent the thermal losses at the walls of the chamber and the erosion effect of gases on the combustion process in the form of a function of the mean density of the stream of gases $G = \rho W$. Here ρ is the density and W is the average velocity of the gases. Thus, it is assumed that in the steady-state, the rate of combustion u and also the surface temperature of the powder T_1 are known functions of the pressure, initial temperature of the fuel T_0, the flow density G, and the temperature of the gases T_2
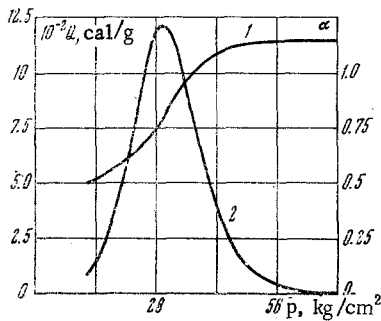
Fig. 1

$$u^\circ = u^\circ(p, T_0, G, T_2), \quad T_1^\circ = T_1^\circ(p, T_0, G, T_2) \tag{1.3}$$

Under nonsteady-state conditions, the pressure and temperature in the chamber are satisfied by the equations

$$V \frac{d}{dt}\left(\frac{p}{RT_2}\right) = \sigma\rho_0 u - \frac{A_0 F_* p}{(RT_2)^{1/2}} \tag{1.4}$$

$$\frac{Vp}{RT_2^2}\frac{dT_2}{dt} = \left(\frac{Q}{c_V T_2} - 1\right)\sigma\rho_0 u - (n-1)\frac{A_0 F_* p}{(RT_2)^{1/2}} - \frac{\sigma_1 q_1}{c_V T_2} \tag{1.5}$$

having the meaning of the laws of conservation of mass and energy. Here t, σ, ρ_0, V, σ_1, and F_* are respectively the time, surface area of combustion, powder density, free volume, internal surface area of the chamber, and critical cross section of the nozzle; c_V is the specific heat of the combustion products at constant volume; R is the gas constant; $n = c_p/c_V$ is the adiabatic index; A_0 is the coefficient of discharge; q_1 is the heat flow density from the combustion products to the walls of the chamber.

If we neglect radiant heat flow in comparison with convective heat flow (the contribution of the various heat loss mechanisms in the chamber is analyzed in detail in [6]), we will have

$$q_1 = D_1 (\rho W)^b (T_2 - T_3) \tag{1.6}$$

where T_3 is the average temperature of the internal surface of the chamber walls; D_1 is a known function of the thermophysical parameters of the combustion products and of the equivalent chamber diameter; b is a constant, ≈ 0.8, for the closed volume conditions. In the future, for approximate estimates, we shall assume T_3 = constant.

The mean density of the gas flow and the discharge of the combustion products through the nozzle are connected by the relation

$$G = \rho W = D_2 \frac{A_0 F_* p}{F (RT_2)^{1/2}} \tag{1.7}$$

where F is the cross-sectional area of the chamber free for the passage of gases; the coefficient D_2 depends on the design of the charge: for end combustion $D_2 \approx 1$ and for channel combustion charges $D_2 \approx 0.5$ under conditions in which the velocity of the gases along the charge is distributed according to a linear law.

Under steady-state conditions the left-hand side of Eqs. (1.4) and (1.5) are vanishing. Given a specified law of combustion, we obtain from Eqs. (1.4) and (1.5) the relations defining the steady-state pressure and temperature in the chamber. In the simplest case, when the thermal losses and erosion are not taken into account, the steady-state temperature and pressure are found from the equations

$$\sigma\rho_0 u^\circ = \frac{A_0 F_* p^\circ}{(RT_2^\circ)^{1/2}}, \qquad T_2^c = \frac{Q(p^\circ)}{c_p}$$

Calculations* for HES 4016 powder show (Fig. 2) that when $F_*/\sigma = 0.0051$, there are three steady-state cycles of combustion, defined by the following values of pressure, temperature, and heat of combustion:

$$p^\circ\ 46\ \text{kg/cm}^2, \quad T_2^c = 2670\ ^\circ K, Q^\circ = 1120\ \text{cal/g}; p^\circ = 32\ \text{kg/cm}^2, T_2^\circ = 2100\ ^\circ K,$$
$$Q^\circ = 880\ \text{cal/g}; p^\circ = 17\text{kg/cm}^2, T_2^\circ = 1320\ ^\circ K, Q^\circ = 555\ \text{cal/g};$$

[In Fig. 2 the points of intersection S_1, S_2, and S_3 of the curves of the specific issue of the gases G_1 (curve 1) and the specific discharge G_2 (curve 2), refer to the combustion surface]. For sufficiently high values of F_*/σ, only the lower cycle exists (in Fig. 2 the point of intersection S_4 of the curves 1 and 3 when $F_*/\sigma = 0.0055$, $p^\circ = 13$ kg/cm^2, $T_2^\circ = 1200^\circ K$, and $Q^\circ = 505$ cal/g); in the case of low values of F_*/σ, it is the upper $(S_5$, the point of intersection of curves 1 and 4 when $F_*/\sigma = 0.0046$, $p^\circ = 69$ kg/cm^2, $T_2^\circ = 2720^\circ K$, and $Q^\circ = 1140$ cal/g).

Figure 3 shows the results of a calculation of the steady-state cycles, taking into account convective heat losses at the walls of the chamber. The thermal losses were determined by the procedure discussed in [6], and the gas flow density was expressed in terms of Pobedonostsev's parameter $\varkappa$ [10]:

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*Here and henceforth, all calculations are carried out for HES 4016 powder, with $T_0 = 25^\circ C$. The required characteristics of the powder are taken from [5, 7-9].
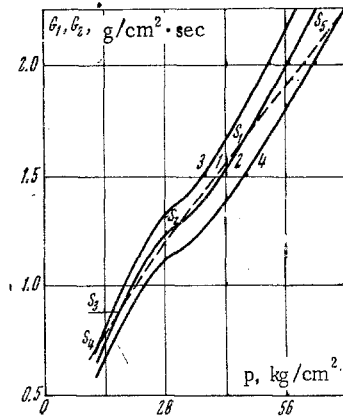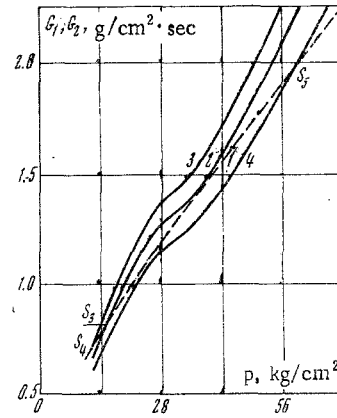
Fig. 2

Fig. 3

$$G° = \varkappa \, \rho_0 u_0° \tag{1.8}$$

where $u_0°$ is the rate of combustion, without taking account of erosion. Curve 1 in Fig. 3 is the specific issue of the gases; curves 2, 3, and 4 are the specific flow rates (discharges) of the products when $F_*/\sigma = 0.0051$, 0.0055, and 0.0046, respectively, and $d_1 = 1$ cm, $\varkappa = 100$, $T_3 = 600°$K, and $\sigma_1 /\sigma = 1$.

In the presence of heat losses, the steady-state cycles are displaced to the side of lower pressures and temperatures in the chamber. In this case the following situation arises: If, for a certain value of F_*/σ, without taking account of thermal losses, only one upper cycle is possible, then if thermal losses are taken into account, the existence of three cycles is found to be possible with the same ratio of F_*/σ; if, without thermal losses, the calculations show the presence of three cycles, then when thermal losses are taken into account and with the same value of F_*/σ, only the lower cycle is possible, etc.

From the disposition of the issue and discharge curves of the combustion products on the Bohr diagram (Figs. 2 and 3), it can be seen that of the three cycles only two are stable — the lower and the upper cycle. The lower combustion cycle usually is not achieved in practice. Obviously, it can be achieved only by means of a special start-up of the half-closed chamber (low initial pressures, high initial powder temperature, and large geometric chamber dimensions). It is possible also that the lower cycle is unstable in relation to the high-frequency region of the spectrum of the combustion product oscillations in the chamber. This problem is not studied in this present paper.

In the future, we shall assume that the nonuniqueness of the steady-state cycle occurs also for the general nonsteady-state case of system (1.4) and (1.5).

We shall mark the quantities which refer to the limiting steady-state cycle, and which are achieved at low pressure and temperature in the chamber, by an asterisk ($p_*°$, $T_{2*}°$, $u_*°$, etc).

We introduce the dimensionless pressure and temperature in the chamber, flow density of the gases, and the rate of combustion:

$$\eta = \frac{p}{p_*°}, \quad \zeta = \frac{T_2}{T_{2*}°}, \quad \lambda = \frac{G}{G_*°}, \quad v = \frac{u}{u_*°}$$

Under steady-state conditions $\eta = \eta°$, $\zeta = \zeta°$, $\lambda = \lambda°$, $v = v°$.

2. Quasisteady-State Approximation

Let us investigate the stability of the steady-state cycle in classical approximation, when a heated layer of powder becomes tuned to perturbations in the gas. In this case, the propagation velocity of the flame is equal to the quasisteady-state rate of combustion z, which in linear approximation is represented in the form

$$z = v° \left[1 + \frac{v}{\eta°}(\eta - \eta°) + \frac{h}{\lambda°}(\lambda - \lambda°) + \frac{q}{\zeta°}(\zeta - \zeta°) \right] \tag{2.1}$$

where the parameters v, h, and q define the degree of dependence of the rate of combustion on the pressure, flow density of the gases, and the temperature in the chamber under steady-state conditions

$$v = \left(\frac{\partial \ln u^\circ}{\partial \ln p}\right)_{T_0, G, T_2}, \quad h = \left(\frac{\partial \ln u^\circ}{\partial \ln G}\right)_{p, T_0, T_2}, \quad q = \left(\frac{\partial \ln u^\circ}{\partial \ln T_2}\right)_{p, T_0, G}$$

We find from Eqs.(1.4) and (1.5), taking into account relations (1.6), (1.7), and (2.1) in linear approximation,

$$t_2 \, (\zeta^\circ)^{1/2} \frac{d\eta}{dt} = B_1, \qquad t_2 \, (\zeta^\circ)^{1/2} \frac{d\zeta}{dt} = B_2 \tag{2.2}$$

Here $t_2 = V/[A_0 F_* (RT_2^\circ)^{1/2}]$ is the relaxation time of the chamber and $a = T_3/T_{2*}^\circ$ is the relative temperature of the walls

$$B_1 = \frac{Q \, (p_*^\circ \eta)}{c_V T_{2*}^\circ} z - n\eta\zeta^{1/2} - \frac{\eta^b (\zeta - a)}{(1-a) \zeta^{b/2}} \left[\frac{Q \, (p_*^\circ)}{c_V T_{2*}^\circ} - n \right]$$

$$B_2 = \left[\frac{Q \, (p_*^\circ \eta)}{c_V T_{2*}^\circ} - \zeta \right] \frac{\zeta z}{\eta} - (n-1) \zeta^{3/2} - \frac{\eta^{b-1} (\zeta - a)}{(1-a) \zeta^{(b-2)/2}} \left[\frac{Q \, (p_*^\circ)}{c_V T_{2*}^\circ} - n \right]$$

The steady-state solutions of system (2.2) are stable in a small region if, as shown in the theory of differential equations [11], the roots of the characteristic equation of system (2.2)

$$y^2 - \left(\frac{\partial B_1^\circ}{\partial \eta} + \frac{\partial B_2^\circ}{\partial \zeta} \right) y + \frac{\partial B_1^\circ}{\partial \eta} \frac{\partial B_2^\circ}{\partial \zeta} - \frac{\partial B_1^\circ}{\partial \zeta} \frac{\partial B_2^\circ}{\partial \eta} = 0 \tag{2.3}$$

have negative real parts. For this, it is necessary and sufficient that the coefficients of Eq. (2.3) should satisfy the Routh-Hurwitz conditions

$$\frac{\partial B_1^\circ}{\partial \eta} + \frac{\partial B_2^\circ}{\partial \zeta} < 0, \qquad \frac{\partial B_1^\circ}{\partial \eta} \frac{\partial B_2^\circ}{\partial \zeta} - \frac{\partial B_1^\circ}{\partial \zeta} \frac{\partial B_2^\circ}{\partial \eta} > 0 \tag{2.4}$$

The steady-state solutions of Eq. (2.2) have the form

$$\eta = \eta^\circ, \qquad \zeta = \zeta^\circ \tag{2.5}$$

Written in the form of Eq. (2.5), these solutions include all the possible steady-state combustion cycles.

Substituting Eq. (2.5) in Eq. (2.4), we obtain the necessary condition of stability of combustion in quasisteady-state approximation:

$$v < \min \left(\frac{\beta_1}{\beta_2}, \beta_3 \right) \tag{2.6}$$

Here

$$\beta_1 = (1-h) \left[N \left(1 - \frac{\alpha}{2} \right) + \frac{a (N-n)}{\zeta^\circ - a} \right] - q \left[\alpha N + (1-b)(N-n) \right]$$

$$N = \frac{Q \, (p^\circ)}{c_V T_2^\circ} \gg n, \quad \beta_2 = \frac{n+N}{2} + \left(\frac{\zeta^\circ}{\zeta^\circ - a} - \frac{b}{2} \right) (N-n)$$

$$\beta_3 = b + (1-b) \frac{n}{N} + \frac{n+1}{2N} + \left(\frac{\zeta^\circ}{\zeta^\circ - a} - \frac{b}{2} \right) \frac{N-n}{N} - \alpha - \frac{(N+1)h}{2N} - \frac{(N-1)q}{N}$$

the parameter α takes into account the dependence of the heat of combustion on the pressure

$$\alpha = \frac{d \ln Q^\circ}{d \ln p}$$

As numerical calculations show, for HES 4016 powder at pressures of $p \geq 12$ kg/cm^2, the value of the parameter α varies over wide limits: $0 < \alpha \leq 1.21$ (in Fig. 1, curve 2); the maximum value of α is reached when $p \approx 30$ kg/cm^2.

We shall express the quantity h in terms of the structural parameters of the chamber and the physicochemical properties of the powder. In the region of positive erosion, according to the usual data [12], the coefficient of erosion $\varepsilon = u^\circ/u_0^\circ$ is represented by a linear interpolated dependence on the dimensionless parameter J [13]

$$J = (\rho W)^\circ \xi_1^{1/2} / (\rho_0 u_0^\circ)$$
$$\frac{u^\circ}{u_0^\circ} = \begin{cases} 1 & \text{for } J^\circ \leq J_* \\ 1 + D_3 (J^\circ - J_*) & \text{for } J^\circ > J_* \end{cases} \tag{2.7}$$
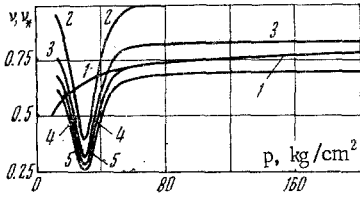
Fig. 4

Here ξ_1 is the drag coefficient

$$\xi_1 = 0.316 \left[\frac{\mu_1}{(\rho W)^\circ d_1} \right]^{1/4}$$

μ_1 is the dynamic viscosity; $d_1 = 4 F / \Pi$ is the equivalent diameter (Π is the perimeter of the chamber cross section free for the passage of the gases). In Eq. (2.7), the value of J depends on the coordinate along the charge. With more accurate data, $D_3 = 0.006$, $J_* = 6.7$ when $6 < J \le 20$ (the interpolated relation $\varepsilon(J)$ is given in [12] over a narrower range of measurement of J).

In the conversion of Eq. (2.7) to the flow density of the gases, we obtain

$$u^\circ = \begin{cases} u_0^\circ & \text{for } J^\circ \le J_* \\ u_0^\circ + D_3 \left[0.562 \left(\frac{\mu_1}{d_1} \right)^{1/8} \frac{(G^\circ)^{7/8}}{\rho_0} - J_* u_0^\circ \right] & \text{for } J^\circ > J_* \end{cases} \tag{2.8}$$

Under steady-state conditions Eq. (1.8) is achieved and therefore from Eq. (2.8) we have

$$h = \frac{7 D_3 D_4 x^{7/8}}{8 (\rho_0 u_0^\circ)^{1/8}} \left\{ 1 + D_3 \left[D_4 \frac{x^{7/8}}{(\rho_0 u_0^\circ)^{1/8}} - J_* \right] \right\}^{-1} \tag{2.9}$$

where

$$D_4 = 0.562 (\mu_1 / d_1)^{1/8}$$

If formulas (2.7)–(2.9) are used for an averaged flow density of the gases, then in place of D_3 it is necessary to take the quantity $D_3 / 2$.

Figure 4 represents the graphical interpretation of the inequality (2.6) for HES 4016 powder when $T_0 = 25^\circ$C. In the calculations, the thermal losses and the dependence of the rate of combustion on the temperature of the gases has not been taken into account ($N = n$, $q = 0$). The curve 1–1 represents the experimental relation $\nu(p)$ [7, 8]. Curves 2–2, 3–3, 4–4, and 5–5 depict the right-hand side of inequality (2.6) ν_* as a function of pressure when $\varkappa = 0$, 100, 175, and 250, respectively, and $d_1 = 50$ cm.

The arms of the curves of $\nu_*(p, \varkappa)$ lying above the line $\nu(p)$ correspond to stable conditions. Taking account of the incomplete heat release gives an upper and a lower limit of combustion with respect to pressure. With increase of $\varkappa$ (for a fixed value of d_1), the upper limit increases but the lower limit falls. The calculations showed that with a reduction of the equivalent diameter, the effect of erosive combustion is intensified. Therefore, in geometrically similar chambers the limiting pressure at which cut-off takes the place of stable combustion is higher in small chambers than in large ones.

Special cases of inequality (2.6) were obtained previously in [3, 4].

3. Low-Frequency Instability

Now let us investigate the stability of the combustion process under nonsteady-state conditions, taking account of the inertia of a heated layer of powder.

Using Zel'dovich's method [1, 14], we proceed from formula (1.3) to the relations

$$u = u (p, f, G, T_2), \qquad T_1 = T_1 (p, f, G, T_2) \tag{3.1}$$

where f is the temperature gradient at the surface of the fuel. Relations (3.1) and also formula (1.1) will be comparable also in nonsteady-state conditions.

We introduce the dimensionless quantities time, coordinate, and temperature in the fuel, and temperature gradient at the surface, by the formulas

$$\tau = \frac{(u_*^\circ)^2}{\varkappa_0} t, \quad \xi = \frac{u_*^\circ}{\varkappa_0} x, \quad \theta = \frac{T - T_0}{T_{1*}^\circ - T_0}, \quad \vartheta = \frac{T_1 - T_0}{T_{1*}^\circ - T_0},$$
$$\varphi = \frac{\partial \theta (0, \tau)}{\partial \xi} = \frac{f}{f_*^\circ}$$

Here $\varkappa_0$ is the coefficient of temperature conductivity of the powder. In linear approximation

$$\eta = \eta^\circ + \eta_1 \psi, \quad \zeta = \zeta^\circ + \zeta_1 \psi, \quad \lambda = \lambda^\circ + \lambda_1 \psi, \quad v = v^\circ + v_1 \psi$$
$$\theta = [\vartheta^\circ + \theta_1 (\xi) \psi] \exp (v^\circ \xi), \quad \vartheta = \vartheta^\circ + \vartheta_1 \psi, \quad \varphi = v^\circ \vartheta^\circ + \varphi_1 \psi$$

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where η_1, ζ_1, λ_1, v_1, $\theta_1(\xi)$, ϑ_1, and φ_1 in the module are much less than the corresponding steady-state values; the function $\psi(\tau)$ defines the change of dimensionless variables with time.

From formulas (3.1), just as in [2-4], relations can be obtained which connect the amplitudes of the rate of combustion, pressure, flow density, temperature in the chamber, and temperature and temperature gradient at the surface

$$(k+r-1)\frac{v_1}{v} = \frac{v(r-1)-\mu k}{\eta^\circ}\eta_1 + \frac{k}{v^\circ\vartheta^\circ}\varphi_1 + \frac{h(r-1)-jk}{\lambda^\circ}\lambda_1 + \frac{q(r-1)-sk}{\zeta^\circ}\zeta_1 \tag{3.2}$$

$$(k+r-1)\frac{\vartheta_1}{\vartheta^\circ} = \frac{\mu(k-1)-vr}{\eta^\circ}\eta_1 + \frac{r}{v^\circ\vartheta^\circ}\varphi_1 + \frac{j(k-1)-hr}{\lambda^\bullet}\lambda_1 + \frac{s(k-1)-qr}{\zeta^\circ}\zeta_1 \tag{3.3}$$

The quantities k, r, μ, j, and s define the degree of dependence of the rate of combustion and the surface temperature on the initial temperature, pressure, flow density, and temperature in the chamber under steady-state conditions

$$k = (T_1^\circ - T_0)\left(\frac{\partial \ln u^\circ}{\partial T_0}\right)_{p,\,G,\,T_2}, \qquad r = \left(\frac{\partial T_1^\circ}{\partial T_0}\right)_{p,\,G,\,T_2}, \qquad \mu = \frac{1}{T_1^\circ - T_0}\left(\frac{\partial T_1^\circ}{\partial \ln p}\right)_{T_0,\,G,\,T_2}$$

$$j = \frac{1}{T_1^\circ - T_0}\left(\frac{\partial T_1^\circ}{\partial \ln G}\right)_{p,\,T_0,\,T_2}, \qquad s = \frac{1}{T_1^\circ - T_0}\left(\frac{\partial T_1^\circ}{\partial \ln T_2}\right)_{p,\,T_0,\,G}$$

For a heated layer of powder, the thermal conductivity equation holds true

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \xi^2} - v\frac{\partial \theta}{\partial \xi} \qquad (0 \geqslant \xi > -\infty) \tag{3.4}$$

(the coordinate origin is located on the burning surface) with the conditions

$$\theta(0,\tau) = \vartheta, \qquad \theta(-\infty,\tau) = 0$$

In linear approximation we have, from Eq. (3.4),

$$\frac{d^2\theta_1}{d\xi^2} + v^\circ\frac{d\theta_1}{d\xi} - \frac{\theta_1}{\psi}\frac{d\psi}{d\tau} - v^\circ\vartheta^\circ v_1 = 0 \tag{3.5}$$

$$\theta_1(0) = \vartheta_1, \qquad \exp(v^\circ\xi)\,\theta_1(\xi) \to 0 \quad \text{for } \xi \to -\infty$$

Assuming that

$$\psi(\tau) = \exp[(v^\circ)^2\Omega\tau], \qquad (\Omega = \omega\varkappa_0/(u^\circ)^2),$$

we find the solution of Eq. (3.5)

$$\theta_1 = \left(\vartheta_1 + \frac{\vartheta^\circ}{v^\circ\Omega}v_1\right)\exp(v^\circ m\xi) - \frac{\vartheta^\circ}{v^\circ\Omega}v_1 \tag{3.6}$$

where

$$m = -1/2 + (1/4 + \Omega)^{1/2}$$

Equation (3.6) permits us to obtain one more relation between the amplitudes of the rates of combustion, temperature, and temperature gradient at the surface

$$\varphi_1 = v^\circ(1+m)\vartheta_1 + \frac{m\vartheta^\circ}{\Omega}v_1 \tag{3.7}$$

Considering Eqs. (3.2),(3.3), and (3.7) as a system of algebraic equations in respect of v_1, η_1, φ_1, ϑ_1, λ_1, and ζ_1, we obtain the formula for the nonsteady-state rate of combustion

$$v = z + \frac{kv^\circ}{1 + rm - k\Psi}\left[\frac{v\Psi - \mu m}{\eta^\circ}(\eta - \eta^\circ) + \frac{h\Psi - jm}{\lambda^\circ}(\lambda - \lambda^\circ) + \frac{q\Psi - sm}{\zeta^\circ}(\zeta - \zeta^\circ)\right] \tag{3.8}$$

where

$$\Psi = 1 - m/\Omega$$

Formula (3.8) is true for any real and complex frequencies. In particular, for low frequencies ($|\omega| \ll (u^\circ)^2/\varkappa_0$), Eq. (3.8) assumes the form

$$v = z + kv^\circ t_1\left[\frac{(v-\mu)}{\eta^\circ}\frac{d\eta}{dt} + \frac{(h-j)}{\lambda^\circ}\frac{d\lambda}{dt} + \frac{(q-s)}{\zeta^\circ}\frac{d\zeta}{dt}\right] \tag{3.9}$$

where $t_1 = \varkappa_0/(u^\circ)^2$ is the relaxation time of the heated layer of powder. It can be seen that in the case of a low-frequency nonsteady-state combustion, the flame propagation velocity at any instant differs from the free quasisteady-state value by the sum of two quantities which are proportional to the scaled pressure, flow density, and temperature of the gases with respect to time. The previously well-known results of [14-16] follows from Eqs.(3.8) and (3.9) as special cases.

By repeating the stability analysis carried out above for the system of equations (1.4) and (1.5), and taking into account relations (1.6), (1.7), and (3.9), we find the conditions for low-frequency stability of the steady-state solution (1.4) and (1.5)

$$kt_1 \, (vN + \beta_5) \, / \, t_2 < 1 \tag{3.10}$$

$$vN - \beta_6 + kt_1 \, [\beta_2 \, (v - \mu + h - j) + \beta_7] \, / \, t_2 < 0 \tag{3.11}$$

$$v < \beta_1 \, / \, \beta_2 \tag{3.12}$$

Here

$$\beta_5 = N \, (h - \mu - j) + (N - 1)[q - s - (h - j)/2]$$
$$\beta_6 = \frac{n + 1 - h(N+1)}{2} + \left(\frac{a}{\zeta^\circ - a} + \frac{b}{2}\right)(N - n) + (1 - \alpha)N - (N - 1)q$$
$$\beta_7 = [\alpha N + (1 - b)(N - n)] \, [q - s - (h - j)/2]$$

In particular, for quasisteady-state approximation $(t_1/t_2 \ll 1)$, Eq. (2.6) follows from Eq. (3.11) and (3.12). However, by taking account of the inertia of the heated layer of fuel it is necessary that inequality (3.10) be satisfied. For this, the parameters of the combustion process, as follows from Eqs.(3.11)-(3.12) and taking Eq. (3.10) into account, should further satisfy the condition

$$v < \min \, (\beta_1/\beta_2, \; \beta_4) \tag{3.13}$$

where

$$\beta_4 = \frac{N \, (\beta_6 - \beta_5) - \beta_2 + \{[N \, (\beta_6 - \beta_5) - \beta_2]^2 - 4N^2 \, [\beta_2 \, (h - \mu - j) - \beta_5\beta_6 + \beta_7]\}^{1/2}}{2N^2}$$

The physical significance of condition (3.13) is the same as in the quasisteady-state case.

For the steady-state law of combustion in the form

$$u^\circ = p^v D \, (T_0) \, H \, (G) \, T_2^{\,q}$$

Eq. (3.10) assumes the form

$$\frac{\varkappa_0\rho_0\sigma RkT_2^{1-q}}{VD \, (T_0) \, H \, (G)} \left[\frac{A_0 F_*}{\rho_0\sigma R^{1/2} D \, (T_0) \, H \, (G) \, T_2^{(1+2q)/2}} \right]^{v_1} (vN + \beta_5) < 1,$$
$$v_1 = (1 + v)/(1 - v) \tag{3.14}$$

The investigation of the conditions obtained for stable combustion, Eqs. (2.6),(3.13), and (3.14) permits a number of conclusions of a qualitative nature to be drawn:

1. The incompleteness of the course of chemical reaction worsens the stable operation of a half-closed chamber.

2. Positive dynamic erosion destabilizes, and negative dynamic erosion stabilizes, the combustion process.

3. For a given charge geometry and chamber volume, reduction of the initial temperature of the powder promotes the appearance of unstable combustion.

4. For a given powder, in order to ensure stable combustion, it is necessary to reduce the ratio of burning surface to free volume.

5. For geometrically similar half-closed chambers, instability is more probable in small-sized chambers than in large-sized chambers.

6. Dependence of the surface temperature on the pressure and temperature in the chamber stabilizes, but dependence on erosion destabilizes, the combustion process.

7. For a given chamber geometry, in the absence of erosion, high-calorific powders burn more stably than low-calorific powders if the power index in the law of combustion v is greater than $(1-4q)/3$, but they burn less stably in the contrary cases.

8. Increase of heat transfer reduces the stability of the combustion process.

9. The dependence of the rate of combustion on the temperature of the gases in the chamber worsens the stable operation of the chamber.

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